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Prandtl-number dependence of interior temperature and velocity fluctuations in turbulent convection

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Temperature and vertical velocity fluctuations are measured in turbulent Rayleigh-Bénard convection at the center of an approximately unit aspect ratio container of cylindrical cross section. Our measurements show that the Rayleigh-number scaling exponent γ of the interior temperature fluctuations (i.e., $\sigma_T/\Delta T \sim \mathrm{Ra}^{\gamma}$) is a strong and nontrivial function of the Prandtl number in the range $2.9 < \mathrm{Pr} < 12.4$. Other measurements at constant $\mathrm{Ra} = 2.0\times 10^9$ show that the interior turbulent fluctuations decrease significantly with increasing Pr; the temperature and velocity fluctuations decrease by about $\sim 45\%$ and $\sim 68\%$ over our range in Pr.

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The study of turbulent Rayleigh-Bénard convection has been vigorously pursued during the past three years [1]. Concerted experimental, theoretical, and computational efforts have sought to explain the global and local properties of turbulent convection. The characterization that is sought by experiments and theories alike, is the variation of the local and global measures as a function of the buoyancy driving parameter, the Rayleigh number Ra, and the Prandtl number Pr. It has long been the expectation that both local and global properties should scale as $\mathrm{Ra}^{\gamma}\mathrm{Pr}^{\delta}$. A comprehensive scaling theory [2,3], partly validated by precise experiments [4–6], has emerged as the leading candidate in describing the variation of global properties in convection cells that are as broad as they are high. These experiments measured the global heat transport and showed that it is better described by the crossover scaling theory of Grossmann and Lohse (GL) [2,3] than by pure power laws.

In the framework of the GL theory, the Ra-Pr parameter space is partitioned into ten regimes depending on the relative boundary layer and bulk dissipations and the thermal and viscous boundary layer length scales. For each regime power-law scaling exponents for the heat transport and the mean large-scale circulation are calculated. Dominant crossover effects from neighboring regimes modify pure power-law scaling. At present, a comprehensive power-law scaling theory for turbulent fluctuations in the different regimes is absent. The theories that address the interior turbulent fluctuations predict pure power-law scalings [7]. A second feature of turbulent convection is that the global heat transport is a very weak function of Pr for $\mathrm{Pr} \gtrsim 1$ and $\mathrm{Ra} \gtrsim 10^6$. Experiments for $3 < \mathrm{Pr} < 35$ and $10^7 < \mathrm{Ra} < 10^{11}$ in approximately unit aspect ratio square and cylindrical cells show a decrease in the heat transport of only 2% [5,8] consistent with the GL theory where significant crossover effects conspire to result in a very weak Prandtl-number dependence. In an independent study, extending the range to higher Pr ~ 1350, it was found that the heat transport $\sim \mathrm{Pr}^{-0.03}$ when described by a power law [6].

In this Rapid Communication, we demonstrate the lack of pure power-law scaling for the interior fluctuations by studying the Ra dependence of the fluctuations at various Pr. For pure power-law scaling of the interior fluctuations $\sim \mathrm{Ra}^{\gamma}\mathrm{Pr}^{\delta}$, the exponents γ and δ should be constants. We find for the temperature fluctuations, however, that $\gamma = \gamma(\mathrm{Pr})$ and $\delta = \delta(\mathrm{Ra})$. In addition, our measurements of the interior fluctuations as a function of Pr at constant $\mathrm{Ra} = 2.0\times 10^9$ show that the fluctuations decay strongly with Pr in contrast to the very weak Pr variation of the heat transport.

We measured the temperature and vertical velocity at the center of a large cylindrical convection cell [9]. Our apparatus is described in detail in Ref. [10]. In brief, the plexiglas cell had diameter $D = 20.9$ cm, height $d = 26.4$ cm, which defines the aspect ratio $\Gamma \equiv D/d = 0.79$. The top and bottom cell boundary were anodized aluminum plates. The top plate was temperature regulated to within $\pm 0.003\,°\mathrm{C}$ and constant heater power was supplied to the bottom plate. A temperature difference ΔT was maintained across the layer, resulting in a Rayleigh number $\mathrm{Ra} = g\alpha d^3 \Delta T/\nu\kappa$, where g is the acceleration of gravity, α is the thermal expansion coefficient, ν is the kinematic viscosity and κ is the thermal diffusivity of the fluid. The measured temperature difference across the fluid was corrected for the thermal conductivity of the aluminum and the anodized aluminum coating. The convection cell was insulated on the vertical and horizontal surfaces with layers of solid foam and fiberglass insulation. The rms temperature fluctuations σ_T were measured using a glass encapsulated semiconductor thermistor which was positioned at the cell center to within 1 mm. Each time sequence consisted of 576–768 K data points sampled at 16.67 Hz. Variation of the signal correlated with variations in room temperature were corrected by subtracting a linear trend over 8 K point intervals. Although this procedure changed the overall magnitude of the rms fluctuations, it did not affect the trend with respect to Ra. We normalize the temperature fluctuations by ΔT. The rms vertical velocity fluctuations σ_V were determined using a commercial Laser Doppler velocimetry system. Each time sequence consisted of 64–128 K data points sampled at 16.67 Hz at the cell center. We normalize σ_V by ν/d.

We varied the mean temperature of our cell between 18.9–60.2 °C when it was filled with distilled water. The corresponding range of $\mathrm{Pr} = \nu/\kappa$ was 7.22–2.99. We ex-

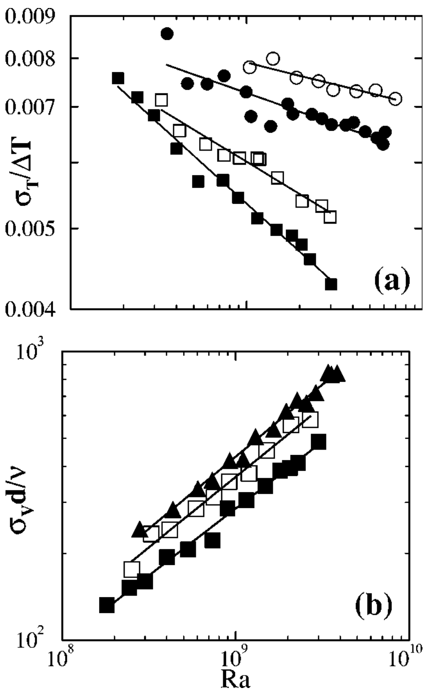

FIG. 1. Normalized rms (a) temperature fluctuations $\sigma_T/\Delta T$ and (b) velocity fluctuations $\sigma_V d/\nu$ vs Ra measured at the cell center for selected Pr. In (a) Pr=3.25 (○), 4.24 (●), 6.49 (□), and 9.99 (■). In (b) Pr=5.46 (▲), 6.48 (□), and 9.99 (■). All lines are power-law fits to the data.

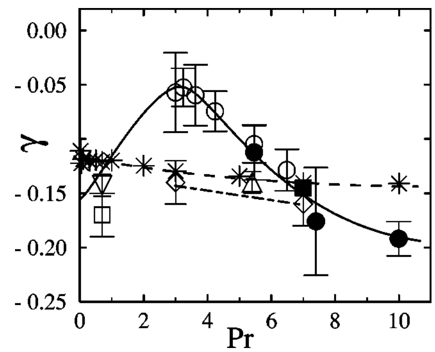

FIG. 2. The power-law scaling exponent γ for the normalized temperature fluctuations at the cell center vs Pr. Present work: in water (○) and in 20% glycerol-water solution (●). Experiments in cylindrical helium cells: ∇ (Refs. [13,14]). Other experiments in cylindrical water cells: △ (Ref. [15]) and ■ (Ref. [16]). Results from a stochastic model are shown in ⋆ (Ref. [17]), and from numerical simulations □ and ◇ (Ref. [18]). The solid line is a guide to the eye illustrating the trend through the data. Dashed lines connect the results from numerical models and simulations with the same geometry.

tended our Pr-number range by using a 20% by mass solution of glycerol in water [11]. In this case a variation of the mean temperature between 22.4–58.9 °C corresponded to a Pr range of 12.32–5.46. As we show below, our data for $5.46 \leqslant \mathrm{Pr} \leqslant 7.22$, which is the Pr overlap between the water and 20% glycerol-water solution are barely distinguishable suggesting that mixture effects that dominate convection close to onset are unimportant at higher Ra [12]. The maximum temperature difference was 9.52 °C, small enough so that the deviations from Boussinesq conditions were insignificant both in water and in 20% glycerol-water solution.

The normalized rms temperature fluctuations $\sigma_T/\Delta T$ are shown in Fig. 1(a) as a function of Ra at various $3.25 \leqslant \mathrm{Pr} \leqslant 9.99$. Assuming that a single power law $\sigma_T/\Delta T \sim \mathrm{Ra}^{\gamma}\mathrm{Pr}^{\delta}$ models the data, we have determined the exponent γ at the various Pr. Since Pr^{δ} is a constant, we expect that γ should be independent of Pr, i.e., the lines in Fig. 1(a) should be parallel. Our data show that the power-law fits have very different exponents ranging from $\gamma(\mathrm{Pr}=3.25)=-0.05 \pm 0.02$ to $\gamma(\mathrm{Pr}=9.99)=-0.19\pm0.02$. The variation in γ implies the lack of pure power-law scaling.

In Fig. 2, we show experimentally measured and numerically computed values of γ at various Pr. The exponent γ is the same whether measured in water ($\mathrm{Pr}=5.47\pm0.01$, $\gamma=-0.10\pm0.02$) or in the 20% glycerol-water solution ($\mathrm{Pr}=5.46\pm0.01$, $\gamma=-0.11\pm0.02$) indicating that turbulent convection in the solution is not modified by mixture or concentration effects. Our measurements are also in agreement with the measurements by other researchers in cylindrical water cells at Pr=5.4 [15] and Pr=7.0 [16]. The results from a stochastic one-dimensional model [17] for Pr<6 and Pr>8 are in disagreement with our measurements. Nevertheless, their results show that the exponent is not constant and varies nontrivially with Pr. Notwithstanding errors in numerical evaluation, it appears that the exponent is a monotonically decreasing function of Pr with $\gamma=-0.33$ at Pr=2750. Given the error bars on the □ and ◇ in Fig. 2, we cannot conclude that the numerical simulations [18] show Pr dependence, however, we cannot discount it either. Two theoretical predictions for pure power-law scaling argue for exponents $\gamma=-1/9$ [7,19] and $\gamma=-1/7$ [7,20,21]. The experimental resolution is insufficient to distinguish between these values and they are consistent with experiments for Pr∼0.7 and for 5<Pr<7. Between 0.7<Pr<5 and for Pr>7, the theoretical predictions are inconsistent with our experimental measurements. The solid line in Fig. 2 illustrates the trend in γ from the experimental data. The maximum in γ suggests a simple scenario in which there are three Pr-number regimes; a small Pr regime where $\gamma \approx -1/7$, an intermediate *Pr* regime $\gamma \sim 0$ and a large Pr regime where $\gamma < -1/5$. Given the error bars on the experimental measurements of γ, we cannot make conclusive statements as to the precise Pr dependence and of the location of the maximum. It is evident, however, that $\gamma=\gamma(\mathrm{Pr})$ which precludes pure power-law scaling.

Assuming that a single power law $\sigma_V d/\nu \sim \mathrm{Ra}^{\lambda}\mathrm{Pr}^{\omega}$ models the normalized rms vertical velocity data, we have determined the exponent λ at Pr=5.46,6.48, and 9.99. The relevant data is presented in Fig. 1(b). We found that $\lambda=0.50 \pm 0.03, 0.49\pm0.04$, and 0.46 ± 0.03, respectively. Independent measurements made by other researchers in a square cell using SF_6 at $d/4$ from the bottom show that for 27<Pr<93, $\lambda=0.43\pm0.02$ and for Pr=190, $\lambda=0.34$ [22]. In our experiments, the variation in λ is insufficient to conclude that $\lambda=\lambda(\mathrm{Pr})$. However, given the small range in Pr and the monotonic trend in the best fit value of λ, we cannot con-

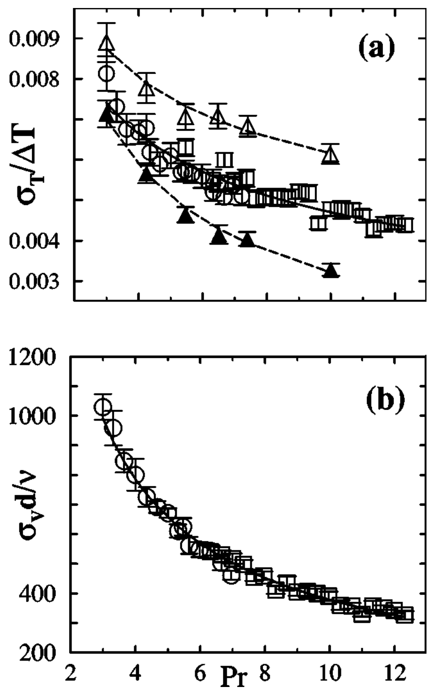

FIG. 3. The normalized fluctuations (a) $\sigma_T/\Delta T$ and (b) $\sigma_V d/\nu$ vs Pr rescaled to $\mathrm{Ra}=2.0\times10^9$ as described in the text. The ○ (□) are measurements made in water (20% glycerol-water solution). In (a) △ (▲) are data rescaled to $\mathrm{Ra}=1.15$ $(3.0)\times10^9$. The △ (▲) symbols are displaced by 0.001 (-0.001) to make them easily visible. All lines are power-law fits to the data.

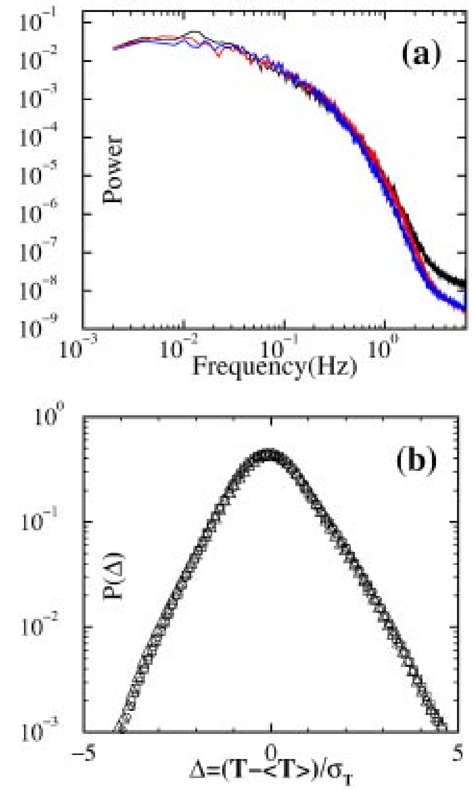

FIG. 4. The (a) frequency power spectra and (b) probability distribution functions of interior temperature fluctuation $(T-\langle T\rangle)/\sigma_T$. The three lines in (a) and symbols in (b) correspond to Pr=3.0,6.5,10.0. All data are at $\mathrm{Ra}\approx3.0\times10^9$.

clude Pr independence either. The case for Pr dependence and therefore against pure power-law scaling can be made more strongly by combining our results with those of the square geometry SF_6 experiment described above. As with the theoretical predictions $\sigma_T/\Delta T$, the predictions for $\sigma_V d/\nu$ call for pure power-law scaling with exponents $\lambda=4/9$ [7,19] and $\lambda=3/7$ [7,20,21]; whereas numerically the experiments and the theories are consistent, the systematic variation of λ with Pr suggests that pure power-law scaling is an inadequate description of the Ra dependence of the interior fluctuations.

In other experiments we systematically varied Pr by varying the mean temperature of the cell at approximately constant $\mathrm{Ra}\approx2\times10^9$. Since Ra was not constant between experiments at different Pr, we rescaled $\sigma_T/\Delta T$ and $\sigma_V d/\nu$ to $\mathrm{Ra}=2.0\times10^9$. The rescaling factor was given by $(2.0\times10^9/\mathrm{Ra}_{expt})^{\gamma,\lambda}$, where γ and λ are the Pr-dependent exponents determined in the experiments at constant Pr discussed above. Suitable interpolation and extrapolation of the γ and λ data were performed to approximate the exponents over $2.9<\mathrm{Pr}<12.4$. In these experiments Ra varied between $(1.73\text{–}2.29)\times10^9$. The resulting rescaling factors for $\sigma_T/\Delta T$ averaged 1% and were bounded by $\pm2.5\%$. For $\sigma_V d/\nu$ the rescaling factors averaged 2% and were bounded below by -3.5% and above by 6.5%. Our Ra-compensation procedure results in corrections to the uncompensated data that are less than the scatter in each set. We plot the rescaled $\sigma_T/\Delta T$ and $\sigma_V d/\nu$ at numerous values of Pr in Fig. 3. We obtain a significant overlap between the data acquired in water and that in 20% glycerol-water solution for $5.46<\mathrm{Pr}<7.22$ for both the temperature and velocity fluctuations, thus reiterating our claim that the effects of solutal concentration gradients are insignificant in turbulent convection.

We fit our data under the assumption of pure power-law scaling: $\sigma_T/\Delta T\sim\mathrm{Ra}^{\gamma}\mathrm{Pr}^{\delta}$ and $\sigma_V d/\nu\sim\mathrm{Ra}^{\lambda}\mathrm{Pr}^{\omega}$. We find reasonable power-law fits to the data with $\delta=-0.38\pm0.04$ and $\omega=-0.80\pm0.03$. Given that the range in Pr is less than a factor of 5, we do not advocate the validity of the fits beyond the range in Pr. In contrast with heat transport, a global property, which varies by a mere 2% over a similar range in Pr, the interior fluctuations are strongly Pr dependent with temperature fluctuations decreasing by $\sim45\%$ and velocity fluctuations by $\sim68\%$. The classical theoretical predictions from pure power-law scaling models call for $\delta=-1/3$ and $\omega=-2/3$ [7,19]. Mixing zone and sheared boundary layer theories predict $\delta=-3/7$ and $\omega=-5/7$ [7,20,21]. Due to the short range of Pr of our data, we cannot make any assertions about the experimentally measured exponents, but, it appears that our measurements are closer to the predictions of the mixing zone and sheared boundary layer theories than to the classical theory.

Analogous to the Pr dependence of the Ra-scaling exponents γ and λ, which was our primary argument for demonstrating that power-law scaling cannot describe the interior fluctuations, measurements of the Ra-dependence of the Pr-scaling exponents δ and ω would further strengthen that conclusion. With much less data than at $\mathrm{Ra}\approx 2.0\times 10^9$, we have measured $\sigma_T/\Delta T$ at six values of Pr and at $\mathrm{Ra}\approx 1.15\times 10^9$ and 3.0×10^9. These data are also shown in Fig. 3(a). The power-law fits give $\delta=-0.34\pm 0.06$ at $\mathrm{Ra}=1.15\times 10^9$ and $\delta=-0.53\pm 0.05$ at $\mathrm{Ra}=3.0\times 10^9$. Hence, we conclude that $\delta=\delta(\mathrm{Ra})$.

Whereas the rms temperature and vertical velocity fluctuations show significant variation with Pr, the frequency power spectra and probability distribution functions (PDFs) appear to be independent of Pr. In Fig. 4, we show the frequency power spectra and PDFs of the temperature fluctuation $(T-\langle T\rangle)$ normalized by the rms value σ_T at $\mathrm{Ra}\approx 3.0\times 10^9$ and for $\mathrm{Pr}=3.0,6.5,10.0$. There is no systematic variation with Pr for both the power spectra and the PDFs. We also find that the power spectra and PDFs for the vertical velocity fluctuations are independent of Pr despite the large variation in the rms values.

We have systematically measured the rms values of the temperature and vertical velocity fluctuations at the center of a cylindrical convection cell of approximately unit aspect ratio. If the power-law scalings $\sigma_T/\Delta T\sim \mathrm{Ra}^{\gamma}\mathrm{Pr}^{\delta}$ and $\sigma_V d/\nu\sim \mathrm{Ra}^{\lambda}\mathrm{Pr}^{\omega}$ are correct models for the interior fluctuations then this separation into Ra-dependent and Pr-dependent parts can be tested. We have determined the Ra-scaling exponents γ and λ at several Pr and the Pr-scaling exponent δ at three values of Ra. Our data show that γ and λ vary with Pr and that δ varies with Ra. This unexpected behavior begs a reexamination of the assumption of the power-law scalings. A possible scenario is that there are several regimes in the Ra-Pr parameter space with different exponents. Hence, long-range crossover effects would allow the exponents to change smoothly as Ra and Pr are varied as has been demonstrated in the experiments and models of heat transport [2–5]. Although such a comprehensive crossover scaling theory for the interior fluctuations is lacking, we can, however, conclude from our data that pure power-law scaling is inadequate. It remains a possibility that power-law scalings may exist at Ra in excess of 10^{10} or for aspect ratios larger than unity, regions not accessed by our experiments. We have also demonstrated that the rms temperature and vertical velocity are sensitive to small changes in Pr in marked contrast to the behavior of heat transport, power spectra, and PDFs.

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[1] L.P. Kadanoff, Phys. Today **54**(8), 34 (2001).
[2] S. Grossmann and D. Lohse, J. Fluid Mech. **407**, 27 (2000).
[3] S. Grossmann and D. Lohse, Phys. Rev. Lett. **86**, 3316 (2001).
[4] X. Xu, K.M.S. Bajaj, and G. Ahlers, Phys. Rev. Lett. **84**, 4357 (2000).
[5] G. Ahlers and X. Xu, Phys. Rev. Lett. **86**, 3320 (2001).
[6] K.-Q. Xia, S. Lam, and S.-Q Zhou, Phys. Rev. Lett. **88**, 064501 (2002).
[7] S. Zaleski, in *Geophysical and Astrophysical Convection*, edited by P. Fox and R. Kerr (Gordon and Breach, New York, 1998).
[8] Y. Liu and R.E. Ecke, Phys. Rev. Lett. **79**, 2257 (1997).
[9] Results are expected to be different for cubic cells due to the strong dependence of the interior fluctuations on lateral geometry. See Ref. [10].
[10] Z.A. Daya and R.E. Ecke, Phys. Rev. Lett. **87**, 184501 (2001).
[11] The temperature dependence of the various physical properties of the glycerol solution were supplied by Sakamoto Yakuhin Kogyo. Co., Ltd.
[12] M.A. Dominguez-Lerma, G. Ahlers, and D.S. Cannell, Phys. Rev. E **52**, 6159 (1995).
[13] J.J. Niemala, L. Skrbek, K.R. Sreenivasan, and R.J. Donnelly, Nature (London) **404**, 837 (2000).
[14] X.-Z. Wu and A. Libchaber, Phys. Rev. A **45**, 842 (1992).
[15] Y.-B. Du and P. Tong, Phys. Rev. E **63**, 046303 (2001).
[16] K.-Q. Xia and S.-L. Lui, Phys. Rev. Lett. **79**, 5006 (1997).
[17] S. Wunsch and A. Kerstein (unpublished).
[18] R.M. Kerr, Phys. Rev. Lett. **87**, 244502 (2001).
[19] R.H. Kraichnan, Phys. Fluids **5**, 1374 (1962).
[20] B. Castaing, G. Gunaratne, F. Heslot, L. Kadanoff, A. Libchaber, S. Thomae, X.Z. Wu, S. Zaleski, and G. Zanetti, J. Fluid Mech. **204**, 1 (1989).
[21] B.I. Shraiman and E.D. Siggia, Phys. Rev. A **42**, 3650 (1990).
[22] S. Ashkenazi and V. Steinberg, Phys. Rev. Lett. **83**, 4760 (1999).